The Behavior of Money Velocity in Low and High Inflation Countries

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Abstract

This paper presents a general equilibrium model of money demand where the velocity of money changes in response to endogenous fluctuations in the interest rate. The parameter space can be divided into two subsets: one where velocity is constant as in standard cash-in-advance models, and another one where velocity fluctuates as in Baumol (1952). The model provides an explanation of why, for a sample of 79 countries, the correlation between the velocity of money and the inflation rate appears to be low, unlike common wisdom would suggest. The reason is the diverse transaction technologies available in different economies.

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1. Introduction

This paper analyzes the long run behavior of money velocity and its relation with the inflation rate. On theoretical grounds, the relation between these two variables is governed by the growth rate of money. At the steady state, countries with higher growth rates of money should present

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higher levels of inflation and larger nominal interest rates. As the opportunity cost of holding money increases, real money demand is reduced and velocity rises. Thus, there should be a strong positive relation between average money velocity and both average inflation and the average rate of money creation. The same reasoning should apply to the volatility of these variables. Countries with more volatile money growth rates should have more volatile inflations and velocities. However, the correlation between the average growth rate of money and the average velocity between the years 1960 and 2000 for a cross-section of 79 countries is just 0.22 while the correlation between the standard deviation of money growth and the standard deviation of velocity over the same sample is 0.40 for those same countries.

I argue that such small correlations arise in the data because the sample pools together countries with dissimilar transaction technologies for which the relation between velocity and inflation (or the growth rate of money) is different. Once countries are sorted by an indicator of transaction costs, the correlation between these variables rises dramatically. To understand why that should be the case and to compute a measure of costs in the financial sector, I develop a general equilibrium version of the models in Baumol (1952) or Tobin (1956) where the velocity of money is determined endogenously and changes in response to fluctuations in the interest rate.

General equilibrium models of the transactions demand for money have been generated in different ways in the literature. Jovanovic (1982) presents a deterministic economy where agents have access to a productive storage technology for capital. Agents want to consume continuously but capital can only be consumed if transferred to the market at a fixed cost. Since capital perishes once removed from storage, money is used to finance consumption between the dates when capital is liquidated. Another route has been taken by the so-called "shopping-time" models of money demand, such as the ones in Den Haan (1990) or Guidotti (1989). In these models, agents value consumption of goods as well as leisure, but transacting goods is a time-consuming activity. Money is demanded because real balances reduce the time needed for transactions. Feenstra (1986) and Marshall (1992) use the same idea but the cost involves goods instead of time. Finally, in liquidity models like Rotemberg (1984), Grossman and Weiss (1983), Alvarez and Atkenson (1997), or Alvarez et al. (2002) agents decide on the composition of their portfolios between cash, needed for transactions, and bonds, bearing an interest rate. However, the number of trips to the bank per period is fixed exogenously in these models.

Romer (1986) presents a general version of the Baumol-Tobin model of money demand where the frequency of financial transactions is determined endogenously. He develops a non-stochastic, continuous time overlapping generations (OLG) economy where agents choose the pattern of money holdings throughout their lives. The model is then used to analyze the Mundell-Tobin effect, the optimum quantity of money, the effect of inflation on consumption and the welfare costs of inflationary finance. However, because of the agent's heterogeneity and the non-recursivity imposed by the OLG structure he is only able to solve the model for the specific case of logarithmic instantaneous utility and no time discount. In fact, as he recognizes, with a different utility function, "... the equations describing equilibrium would therefore be extremely complicated, and the analysis of those equations extremely impossible."

Cash-in-advance (CIA) models can also be seen as general equilibrium versions of the Baumol-Tobin model with the following transactions technology: the first trip to the bank is free and the subsequent ones are prohibitively expensive. Thus, the agent only goes to the bank once. This transactions technology is the basis for the unitary money velocity prediction of the simplest versions of CIA models. Two extensions have been developed in the literature to overcome this unrealistic feature. One is the introduction of a precautionary motive to hold currency as in Svensson (1985) by changing the information structure of the model. With this new setup, information about the current state of the economy is revealed only after the agents make their decisions on money holdings. Once uncertainty is resolved there will be states of the world where all cash balances are not spent and the velocity of money can vary. The second extension involves the introduction of a second good whose consumption does not need to be paid for with cash as in Lucas and Stokey (1987). In these models, money velocity changes because the relative proportion of consumption in cash goods (goods whose consumption has to be paid for with cash) and in credit goods (goods whose consumption does not need to be purchased with cash) changes over time depending on the economic fundamentals.[1] However, as it is shown in Hodrick et al. (1991), neither interpretation can explain the variability of money velocity observed in the data.

The model developed here contributes to the literature outlined above. First, it extends conventional CIA models by allowing money velocity to be different than one. In fact, the CIA model of Lucas (1980a) appears as a particular case for a subset of the parameter space.[2] In my economy, agents are able to economize on their cash holdings by changing the number of trips to the bank in response to changes in the interest rate. As in CIA models, markets open sequentially. An asset market opens at the beginning of the period after the state of the economy is realized. In this market, the agent deposits his initial wealth in an illiquid bond and plans to make withdrawals of money that he will use to finance consumption during that period. There is a fixed transaction cost per withdrawal (except for the first one which is free) equal to a proportion ϕ of current nominal output. Interest rates are determined at the beginning of the period and are paid at the end of the period on average bond holdings. During the second subperiod, a product market opens and the agent finances consumption with the successive money withdrawals. For values of the parameter ϕ that are small enough, the Baumol's inventory-theoretic considerations that are responsible for variations in money velocity in response to endogenous changes in the interest rate will appear here too. So, unlike CIA models, velocity is endogenous here. Contrary to “shopping time” models, the transaction technology is made explicit which helps in the calibration and interpretation of the parameters of the model.

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[1] Schreft (1992) develops an overlapping generations model with a physical setup similar to the one presented here where the mix of cash and credit goods is determined endogenously.

[2] The model in Svensson (1985) can also be included as a particular case if the timing of markets is reversed. See Rodríguez Mendizábal (1998) for details.

Finally, unlike Romer (1986), the inventory-theoretic aspects of money demand do not add any complication to the solution so the model is as simple to solve as typical dynamic models in this literature. The reason is that the transactions technology here is static and it is still possible to preserve the recursive nature of these types of models.

Furthermore, the model gives a simple explanation for why the mean and standard deviation of velocity have a low correlation with the respective moments of inflation and the growth rate of money. The correlation between these variables should be high for countries with similar transaction technologies, the parameter ϕ in the model, while in the data all countries are pooled together. So, I use the model to compute this parameter for each country. Once the sample is sorted according to this measure of transaction costs, the correlation between velocity and inflation increases dramatically both for the first and second moments.

The paper is organized as follows. Section 2 describes the model. Section 3 presents the empirical application and section 4 concludes and addresses future applications.

2. A Transaction Demand for Money

The setup of this economy is similar to Lucas (1980a). Time is continuous and is divided in fixed intervals called periods or days. Uncertainty, namely the output to be produced and the injection of new money, is resolved at the beginning of these periods before decisions are taken. There is a continuum of infinitely lived households indexed by h, $h \in H = [0,1]$. Every day, a good is produced in J different varieties or colors indexed by j = (1, …, J), with items of each color produced and sold in spatially separated factories-stores. These firms are distributed evenly around a circle of length one in J locations or villages. Contemporaneous preferences of the h-th household over all possible varieties of the good are summarized by the function

$$U\left[c_t^h(1),\ldots,c_t^h(J)\right] = u\left[\prod_{j=1}^{J}\left(c_t^h(j)/\alpha_j\right)^{\alpha_j}\right], \tag{1}$$

with $c_t^h(j)$ being consumption of household *h* of variety *j* during period *t*. The function u:$R_+ \rightarrow R$ is continuous, twice differentiable, strictly increasing and strictly concave. Also $\sum_{j=1}^{J} \alpha_j = 1$ and $\alpha_j > 0$ all *j*. Define $C_t^h(j) = \sum_{j=1}^{J} c_t^h(j)$. Since in equilibrium relative prices will be one across colors, it turns out that

$$c_t^h(j) / C_t^h = \alpha_j,$$

so that

$$U\left[c_t^h(1), \ldots, c_t^h(J)\right] = u\left[C_t^h\right]. \tag{3}$$

Each household is composed of a worker-shopper pair. The couple lives in the location where the worker goes to work. This means that a proportion α_j of the households lives in location *j*. They own the firm in that location. The worker is endowed with a unit of labor, which yields y_t^j units of the particular color produced at that village. The shopper, on the other hand, dedicates his time to visit all the shops around the circle to buy the other varieties of the good. It is assumed that he can only move clockwise and takes one period to complete the trip. If, as in Lucas (1980a), there is limited communication among the stores so it is costly for them to verify each shopper's credit history, there is an incentive in this economy to use currency as a means to save on information costs.[3]

The introduction of money in this model is as follows. Assume that in order to distribute money to the households, a bank is created in each location. Firms and households have accounts at the bank in their home location. The period begins with shoppers at the bank in their respective

[3] The different varieties of the good only play the role of motivating a demand for money as an information saving device. With preferences as in (1) and relative supplies specified above, relative prices will always be equal to one independently of any monetary arrangement as long as the different varieties are treated symmetrically. Thus, the only variable of interest for the maximization problem of the household is its aggregate consumption c_t^h.

villages. Household h starts with initial nominal wealth W_t^h. Then an asset market opens where the shopper decides on an initial deposit in the bank plus the number and sizes of money withdrawals that he will make while going around the circle and will use to finance his consumption in that period. These withdrawals are charged to the initial bond holdings. Each withdrawal (except for the first one, which is free) costs a fee Φ_t to be specified below. These fees are paid to the bank at the end of the period. The shopper keeps visiting firms around the circle and buying the consumption goods until he runs out of money in some store. Then he visits the bank in that location and claims the next withdrawal. After withdrawing the money the shopper returns to the product market to buy goods. He will continue to do this the number of times contracted previously. As the agents buy the goods, the firms take the money back to the bank and deposit it there. Firms also face cash management costs, which are symmetric to the ones born by households. Interest is paid at the end of the period to households and firms on average deposits.

The nature of the fee is explained as follows. When an agent wants to make a withdrawal other than the first one, he will do it in a bank where he does not have an account. In that case, his bank will need to hire a proportion ϕ of workers from the shopper's location. These workers will do all the necessary bookkeeping for that transaction. Since banks know from the beginning of the period how many withdrawals the shoppers from their villages are going to make, they can also hire these workers at the beginning of the period. This is the origin of the fee, which serves to pay the wages of the workers at the bank. At the end of the period shoppers pay the fees to banks, banks pay interest on average deposits to firms and households and wages to workers, and firms pay wages to workers and dividends to households. New money enters the system through a monetary transfer H_{t+1} that is given to households at the beginning of the next period, and the process starts all over again.

2.1. The Household's Problem

Each period, household *h* has to choose how much to consume and how to finance this consumption, that is, how many times to go to the bank and how much money to withdraw in each trip. Assume the shopper spends at a constant rate around the circle. Let N_t^h be the contracted number of trips to the bank within period *t*. Rodríguez Mendizábal (1998) shows that, in this setup, trips to the bank will be evenly spaced so that, at any time *t*, the withdrawals are of the same size. This is because of the constant interest rate and consumption rate throughout the period. Denote by M_t^h the size of any of these withdrawals. If B_t^h represents initial bond holdings, the financial constraint for the asset market is

$$B_t^h + M_t^h \leq W_t^h, \tag{2}$$

while average account balances equal

$$\overline{B}_t^h = B_t^h - \frac{N_t^h - 1}{2} M_t^h.$$

In the product market the agent uses the money obtained from the trips to the bank to finance consumption of that period, that is,

$$P_t C_t^h \leq N_t^h M_t^h, \tag{3}$$

with P_t being the nominal price level. Finally, wealth evolves as

$$W_{t+1}^h = W_t^h + H_{t+1}^h + L_t^h + D_t^h + i_t \overline{B}_t^h - P_t C_t^h - \left(N_t^h - 1\right)\Phi_t, \tag{4}$$

where H_{t+1}^h is the exogenous money transfer, L_t^h represents wages received either from firms or banks, and D_t^h is equal to dividend earnings.

Given the processes for $\left\{P_t, i_t, H_{t+1}^h, L_t^h, D_t^h, \Phi_t\right\}_{t=0}^{\infty}$ and initial wealth W_0^h, the household *h* chooses sequences $\left\{C_t^h, M_t^h, B_t^h, N_t^h\right\}_{t=0}^{\infty}$ to maximize expected lifetime utility

$$E_0\left[\sum_{t=0}^{\infty} \beta^t u\left(C_t^h\right)\right],$$

where $0 < \beta < 1$, subject to (2), (3) and (4) as well as nonnegativity constraints,

$$C_t^h \geq 0, M_t^h \geq 0, \tag{5}$$

and the condition that he visits the bank at least once each period,[4]

$$N_t^h \geq 1. \tag{6}$$

2.2. The Firm's Problem

The problem faced by each firm is as follows. They just produce a color of the good, receive the interest on average deposits from banks, and pay wages to workers and dividends to households living in their location. They also have to decide when to go to the bank to deposit the money they receive from shoppers. Since they always deposit in a bank in their location they do not have to pay fees but instead firms have to dedicate a fraction ϕ of the labor force to cash management duties.[5] Although there is one firm per location, it is assumed that potential entrants drive existing firms' profits to zero. Therefore, for firm j it must be the case that,

$$i_t \overline{B}_t^j + P_t y_t^j \int_{A^j} \left[1 - \left(N_t^j - 1\right)\phi - \left(N_t^h - 1\right)\phi\right] dh = \int_{A^j} L_t^h \left[1 - \left(N_t^h - 1\right)\phi\right] dh + \int_{A^j} D_t^h dh, \tag{7}$$

where N_t^j is the number of firm's deposits in the bank, $\overline{B}_t^j$ is the firm's average account balance

$$\overline{B}_t^j = \frac{N_t^j - 1}{2} M_t^j,$$

[4] For tractability the integer constraint on N_t^h is ignored throughout the paper. This should be seen as approximating a step function with a differentiable one to ease computations. The reason for this approximation is because N_t^h will later be related to the velocity of money. The results described below do not hinge on this simplification.

[5] This assumption just makes the cash-management problem of the firm symmetric to the one of the household. From the discussion of the household's problem, the firm will also space the deposits evenly within the period.

and A^j is the mass of people living in location j ($j = 1, \ldots, J$),

$$A^j = \left[\sum_{k=0}^{j-1} \alpha^k, \sum_{k=0}^{j} \alpha^k\right],$$

with $\alpha^0 = 0$. The second term on the left side of (7) represents total production after labor has been hired by banks as well as used for the firm's cash management.

2.3. The Bank's Problem

The bank in location j makes interest payments on average deposits by the firm in that location, $\overline{B}_t^j$, and by households, $\overline{B}_t^h$, $h \in A^j$. The bank also receives fees from shoppers and pays wages to workers. As with firms, a zero-profit bank in location j satisfies

$$\Phi_t \int_{A^j} \left(N_t^h - 1\right) dh = i_t \int_{A^j} \overline{B}_t^h dh + i_t \overline{B}_t^j + \phi \int_{A^j} L_t^h \left(N_t^h - 1\right) dh . \tag{8}$$

The left side of expression (8) represents the fees collected from households. The right side starts with the total interest paid to households, followed by interest paid to firm j on average deposits and finally, the third term refers to total wages to bank workers.

2.4. The Representative Agent Economy

Under the assumption that all agents are identical, it is possible to look at the representative agent version of this economy. Firms' decisions about when to deposit money in the bank are symmetric to households' decisions about when to withdraw it so that they will make the same number of trips, that is,

$$N_t^j = N_t^h = N_t .$$

for all j ($j = 1, \ldots, J$) and $h \in H = [0, 1]$. Also, in the representative agent economy, wages and dividends will be equal to

$$L_t + D_t = P_t Y_t + i_t \overline{B}_t^a - \left(N_t - 1\right) \phi P_t Y_t . \tag{9}$$

where $Y_t = \sum_{j=1}^{J} y_t^j$, and $\overline{B}_t^a$ is the firms' aggregate average account balances. These are taken as given by the agents when making their decisions. Notice that, in the aggregate, average account balances in the bank are zero since deposits by firms correspond to withdrawals by agents. However, at the individual level, there are incentives to go several times to the bank in order to save on foregone interest earnings. Also, it is easy to see that the fee will be equal to the value of labor services, or $\Phi_t = \phi P_t Y_t$. This is because the opportunity cost for a worker of being hired by a bank is the revenue from working in the firm and producing $P_t Y_t$ units of the good.

There are two sources of uncertainty in this endowment economy. Real output (Y_t) grows at an exogenous, stochastic rate γ_t. Money supply (M_t) is also exogenous and growing stochastically at a rate μ_t. Thus,

$Y_t = (1 + \gamma_t)Y_t$, and $M_t = (1 + \mu_t)M_t$.

The rates γ_t and μ_t are known at the beginning of period t. Let (Θ, Z) be a measurable space. Uncertainty is then characterized by the shock $\theta_t = [\gamma_t, \mu_t]' \in \Theta$. The set Θ is assumed to be compact. This shock follows a first-order Markov process with transition probability function Q: $\Theta \times Z \rightarrow [0, 1]$ assumed to be monotone and to satisfy the Feller property.

2.4.1. Definition of Equilibrium

Assuming $i_t > 0$, so that the generalized CIA constraint (3) is binding, we can substitute N_t by

$$N_t = \frac{P_t C_t}{M_t^d}$$

in (4) and (6). So, the problem of the household is to choose sequences for consumption (C_t), money holdings (M_t^d), withdrawals to make (N_t) and bonds demand (B_t^d) to maximize his expected utility

$$E_0\left[\sum_{t=0}^{\infty} \beta^t u\left(C_t^h\right)\right]$$

given his initial wealth (W_t), the evolution of prices (P_t, i_t, L_t, D_t) and of the endowment processes (summarized by h_t). The restrictions of this maximization problem are the asset constraint

$$B_t^d + M_t^d \leq W_t\,, \tag{10}$$

the condition that the agent visits the bank at least once

$$\frac{P_t C_t}{M_t^d} \geq 1\,, \tag{11}$$

the nonnegativity constraints (5) an the evolution of wealth

$$W_{t+1} = W_t + H_{t+1} + L_t + D_t + i_t \overline{B}_t^d - P_t C_t - \left(\frac{P_t C_t}{M_t^d} - 1 \right) \Phi_t\,, \tag{12}$$

where $L_t + D_t$ is given by (9).

Definition 1. A *competitive equilibrium* is a sequence of allocations $\{C_t, M_t^d, B_t^d, N_t\}_{t=0}^{\infty}$ and prices $\{P_t, i_t, L_t, D_t\}_{t=0}^{\infty}$ such that given the evolution of the exogenous shocks summarized by the stochastic process $\{\theta_t\}_{t=0}^{\infty}$,

1. wages and dividends satisfy (9),

$$L_t + D_t = P_t Y_t + i_t \overline{B}_t^a - (N_t - 1)\phi P_t Y_t\,,$$

2. the allocation sequence solves the consumer's problem given prices, and
3. markets clear, that is, goods[6]

$$Y_t = C_t + (N_t - 1) 2\phi P_t Y_t\,, \tag{13}$$

money

$$M_t^d = M_t\,, \tag{14}$$

and bonds

[6] In this representative agent economy, each trip to the bank implies two trips in the economy of the previous section: one by the household and one by the firm.

$$B_t^d = 0. \tag{15}$$

The first order conditions (FOCs) of this problem are,

$$\frac{u'(C_t)}{P_t} + \frac{\lambda_t}{M_t} = \beta\left[1 + \frac{i_t}{2} + \frac{\phi P_t Y_t}{M_t}\right] E_t(\delta_{t+1}), \tag{16}$$

$$\delta_t + \frac{\lambda_t P_t C_t}{(M_t)^2} = \beta\left[1 + \frac{i_t}{2} + \frac{P_t C_t}{(M_t)^2}\phi P_t Y_t\right] E_t(\delta_{t+1}), \tag{17}$$

$$\delta_t = \beta(1 + i_t) E_t(\delta_{t+1}), \tag{18}$$

$$\left[\frac{P_t C_t}{M_t} - 1\right]\lambda_t = 0;\ \lambda_t \geq 0, \tag{19}$$

plus the constraints (10) and (12) where δ_t and λ_t are the Lagrange multipliers associated with (10) and (11), respectively, and $E_t(\bullet)$ represents the conditional expectation with respect to the information set in t. Notice I have already substituted for the equilibrium values of money holdings. These conditions have the usual interpretations. For example, the left-hand side of (16) is the total marginal utility derived from an increase in consumption. The first element is the direct increase in the utility function. Also, an increase in consumption relaxes constraint (11). The second element refers to the value of this effect. The right-hand side includes the marginal costs associated with increasing consumption. Since money has to be used to consume, these costs include the foregone interest from the reduction of the average bond holdings plus the transaction costs of new trips to the bank. The rest of conditions have similar interpretations.

2.4.2. Discussion of the Equilibrium

Conditions (13) to (19), plus constraint (12) form a system of eight equations in eight unknowns. These unknowns are two prices, P_t and i_t, three allocations, C_t, M_t^d, and B_t^d, two multipliers, λ_t and δ_t, and next period's wealth, W_{t+1}.

Expressions (16), (17) and (19) yield

$$\frac{u'(C_t)}{P_t} = \delta_t - \beta \frac{\phi P_t Y_t}{M_t}(N_t - 1)E_t(\delta_{t+1}).$$

When the constraint (11) is binding and $N_t = 1$, the model behaves as a standard CIA model and the marginal utility of consumption is equal to the marginal utility of wealth. However, when that constraint is not binding and $N_t > 1$, the marginal utility of consumption is larger than the marginal utility of wealth. This is because in this case not all increases in income can be transferred to increases in consumption since a proportion is lost in transaction costs. In this case, the Baumol's square-root formula

$$\frac{M_t^d}{P_t} = \sqrt{\frac{2C_t \phi Y_t}{i_t}}$$

can be obtained from (17) and (18).

Proposition 1. Let the utility function be of the form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \ \sigma \geq 0.$$

Let v_t be the income velocity of money defined as

$$v_t = \frac{P_t Y_t}{M_t}.$$

Define $\bar{\phi} > 0$ as

$$\bar{\phi} \equiv \frac{1}{2}\left[\frac{1}{\beta\Delta} - 1\right] \tag{20}$$

where

$$\Delta = \min\left\{E_t\left[\frac{1}{(1+\mu_{t+1})(1+\gamma_{t+1})^{\sigma-1}}\right]\right\}.$$

Then, for all $\phi \geq \bar{\phi}$ and $\theta_t \in \Theta$,

$v_t = 1$.

Proof. See Appendix A.

This result shows that the CIA model in Lucas (1980a) is a limiting case of this model since for a ϕ that is high enough the agent will go to the bank just once. It is often said that we can look at CIA models as economies with the following transaction technology. The first trip to the bank is free and the next ones are prohibitively expensive which is what makes the agent go only once to the bank. Proposition 1 states in an objective way what "prohibitively expensive" means.

Finally, from the definition of velocity, inflation is equal to

$$1+\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{(1+\mu_t)v_t}{(1+\gamma_t)v_{t-1}}.$$

Clearly, in the standard CIA version of the model when $v_t = 1$ for all θ_t, inflation is just equal to the ratio of $(1 + \mu_t)$ over $(1 + \gamma_t)$.

3. Empirical Application

3.1. Data

For the empirical application of the model I collected annual data from 1960 to 2000 on real GDP, nominal GDP, GDP\ deflator, population, and M1 from the World Bank Development Indicators Database. Whenever a particular series was not available for some country I used the IMF International Financial Statistics Database. To have enough information for each economy, I only used countries with at least 30 years of data. This produced a dataset of 79 countries. Geographically, this sample includes 10 countries from Europe, 26 from America, 13 from Asia-Oceania and 30 from Africa-Middle East. Appendix B includes information about these series.

With this data I computed the growth rates of per capita GDP (γ_t in the model), the growth rates of per capita money holdings (μ_t), inflation (π_t) and velocity of money (v_t) for each country and each year in the sample. Then I computed the average and standard deviation of these variables for each country over the sample period. Thus, each country is characterized by eight numbers corresponding to its time average and standard deviation of its growth rate of money, growth rate of per capita GDP, inflation and velocity. In this manner all the information in my dataset is summarized in eight series, four averages and four standard deviations, of 79 observations, the countries in the sample.

Table 1 collects the correlation of the growth rate of money with velocity and inflation both for the averages as well as the standard deviations. The first line corresponds to all the countries in the sample. We observe that inflation and the growth rate of money are highly correlated both for the averages as well as the standard deviations. This is just another example of the quantity theory of money illustrated in Lucas (1980b). However, the growth rate of money and velocity are not as correlated as expected. For averages the correlation is 0.2275 while for standard deviations is slightly larger, 0.4066. One possible reason for these low values is that we are mixing very diverse countries. The rest of lines in the table divide the sample in four areas: Industrial countries, Latin America, Asia and Africa.[7] Although the correlations improve for Industrial countries, results are mixed for the others areas.

Figure 1 displays a scatter plot of the average velocity versus the average growth rates of money for the 79 countries. We observe there are five outliers (Argentina, Bolivia, Brazil, Peru and Nicaragua). These countries may be causing the low measures of correlation documented above. However, after taking out these countries, the correlation between velocity and the growth rate of money is still 0.3276 for averages and 0.5466 for standard deviations.

[7] Appendix B defines each of the areas.

Another possibility of these low correlations could be that I am not controlling for the growth rate of real output. After all, velocity should be a function of this variable together with the growth rate of money. Column (1) of Table 2 presents the results of an OLS regression of the average velocity on the average growth rates of money and output for all the countries in the sample. It shows how little explanatory power these two variables have in explaining the variability of velocity across countries, with an R-squared of 0.115. The average growth rate of money is not even significant (*t*-statistic of 1.57). This table also includes the results for the standard deviations in column (3). The same general conclusion of little explanatory power applies. Table 3 shows the corresponding results for inflation. Here we see how inflation is fundamentally explained by the growth rate of money both for its average as well as its standard deviation. Notice columns (1) and (3) on this table are the regressions derived from the standard CIA model.

3.2. The Behavior of Velocity

This section provides an interpretation as of why the data shows a low correlation between velocity and the growth rate of money. This correlation should be high for countries with similar transaction technologies, the parameter ϕ in the model. However, there should not be such a close relation for countries with different transaction costs. Below, I will use the model to calibrate a value for ϕ for each country. Once we condition on this parameter the correlations between velocity, inflation and money growth rates increase dramatically both for averages as well as for standard deviations.

There are three parameters in the model: the time discount, β, the inverse of the elasticity of intertemporal substitution, σ, and the transaction cost, ϕ. I fixed the preference parameters for all countries to $\beta = 0.99$ and $\sigma = 1$ and computed ϕ for each country to match its average velocity over the sample period. Since the model cannot be solved analytically, as it is usual within this stochastic dynamic general equilibrium literature, I approximated the solution by using the Parameterized Expectations Algorithm (PEA) explained in Den Haan and Marcet (1990). The way the PEA solution method works is by approximating the expectation in (16), (17), and (18) with a particular functional form.[8]

Countries were ordered according to the calibrated parameter ϕ and clustered in four groups through a *k*-means procedure. This algorithm starts with an initial partition of the data in *k* clusters. Distances of each data point to the mean of each group are computed and observations are reassigned to the closest group. This process is repeated until no element changes groups.[9] The estimated values of ϕ for each country sorted by groups is included in Appendix C. Table 4 presents the correlations between the growth rate of money with velocity and inflation for each of the groups.[10] The third column shows the average value of the transaction cost ϕ for each group. As the countries were ordered by the parameter ϕ so are the groups. While the correlations between the money growth rate and inflation are still high, the correlations with velocity increase significantly as compared with the numbers for the whole sample. Furthermore, column (2) of Table 2 repeats the regression of the average velocity on the average growth rates of money and output for all the countries in the sample including the calibrated ϕ as an explanatory variable. Column (4) includes the results for the standard deviations. This measure of transaction costs is statistically significant in both regressions. The *R*-squared of the regressions are improved as well as the *t*-statistics for the other explanatory variables. Table 4

[8] In order to simulate the exogenous process $\{\theta\}$, I use the bootstrap procedure of Politis and Romano (1994) on data for the growth rates of money and output for each country.

[9] See, among others, Jain and Dubes (1988). I also did the computations with 3 and 5 groups finding similar results.

[10] Group 4 is composed of Nicaragua only. The calibration of ϕ for this country was very far away from the rest so it was never clustered in any of the other groups.

shows that the calibrated ϕ is also significant for inflation improving the results of standard CIA models.

Figures 2 to 4 provide the scatter plot of average velocity and average growth rate of money for the first three groups. Several conclusions are drawn from these figures. First, each group separately presents a positive relation between velocity and the growth rate of money. Second, groups are parallel to each other so that, as the average transaction cost parameter ϕ of the group increases, the cluster moves to the northeast in the graph. The reason is simple. As the parameter ϕ gets larger, agents find it more costly to turn around money and thus, velocity is reduced for the same growth rate of money. This is why, when all countries are pooled together, correlations were smaller for the sample as a whole. Third, the calibration exercise separates the outliers in the sample. Four of them (Argentina, Bolivia, Brazil and Peru) are now included in group 3 while Nicaragua forms a group by itself.

4. Conclusions

This paper develops a simple general equilibrium model of money demand where the velocity of money is determined endogenously as in Baumol (1952) or Tobin (1956). Also, it is shown that versions of this model nest standard cash-in-advance models as particular cases for a subset of the parameter space.

Making velocity endogenous in this model comes at no computational cost since it means only adding a simple static problem to the standard general equilibrium CIA models. Despite its simplicity, however, the model performs surprisingly well when approximating the relation between the growth rates of money and output with inflation and velocity. In particular, the model helps understanding why the mean and standard deviation of velocity have a low correlation with the respective moments of inflation and the growth rate of money. The correlation between these variables should be high for countries with similar transaction technologies, the parameter ϕ in the model, while in the data all countries are pooled together. Once the sample is sorted according to this measure of transaction costs, the correlation between velocity and inflation increases dramatically both for the first and second moments. Furthermore, the paper shows that the calibrated measure of transaction costs is also significant in the relation linking the money growth rate with inflation.

In this model, money is neutral. Short-term nonneutralities have been recently analyzed in the literature with limited participation and sticky price models. Typically, however, they fail to generate enough volatility for the velocity of money. As Christiano (1991) points out with respect to limited participation models, their empirical implications "... may be improved by incorporating a more flexible transaction model of money demand, since presumably this would increase the interest sensitivity of velocity." The economy described in this paper embodies such a technology in a simple way so it could perform this task. In particular, it could be easily combined with limited participation or sticky price models. This should improve the response of money velocity to changes in the interest rate without modifying significantly other implications of these models. The reason is that the transaction technology leaves practically unaffected the intertemporal allocations and pricing functions of these economies.

On the other hand, unlike "shopping-time" models that introduce ad-hoc transaction functions, the transactions technology included in the paper is made explicit. This feature could be useful in some applications especially in international economics where assumptions about that transactions function may not be very intuitive. One such application is the analysis of the efficiency gains derived from a monetary union. Most of the recent literature on the topic is associated with the EMU and points out two main direct benefits: the elimination of transaction costs associated with exchanging currency and the suppression of exchange rate uncertainty.[11] Rodríguez Mendizábal (2001) is an attempt in tackling this problem. It presents a two-currency version of the model in this paper and shows how it can be helpful in this research by incorporating, in a simple way, an explicit transactions technology in a CIA model.

A. Proof of Proposition 1

First, define the normalized multipliers ξ_t and η_t as

$$\xi_t = \delta_t M_t y_t^{\sigma-1} \text{ and } \eta_t = \lambda_t y_t^{\sigma-1}.$$

For the CRRA utility

$$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \ \sigma \geq 0 \ ,$$

the equilibrium conditions may be rewritten as the following system on the variables v_t, ξ_t, and η_t:

$$\frac{(1+2\phi v_t)^\sigma}{(1+2\phi)^\sigma v_t} = \beta\left(1+\phi v_t + \phi\frac{(1+2\phi)v_t^2}{1+2\phi v_t}\right)E_t\left[\frac{\xi_{t+1}(1+\gamma_{t+1})^{1-\sigma}}{1+\mu_{t+1}}\right] - 2\eta_t ,$$

$$\xi_t = \beta\left(1+2\phi\frac{(1+2\phi)v_t^2}{1+2\phi v_t}\right)E_t\left[\frac{\xi_{t+1}(1+\gamma_{t+1})^{1-\sigma}}{1+\mu_{t+1}}\right] - 2\eta_t , \tag{21}$$

and

$$(v_t - 1)\eta_t = 0; \ \eta_t \geq 0 .$$

Notice that $\xi_t = 1$ whenever $v_t = 1$. Then, we need to find the value of ϕ such that $\eta_t > 0$, for all possible θ_t. For that, use (21) and after plugging these values obtain the condition

$$2\eta_t = \beta(1+2\phi)E_t\left[\frac{(1+\gamma_{t+1})^{1-\sigma}}{1+\mu_{t+1}}\right] - 1 > 0 .$$

[11] For evaluations of the main benefits and costs see, among others, Commission of the European Communities (1990), Fratiani and von Hagen (1992) and Gross and Thygesen (1992).

Operating on this expression it is easy to show that the value of ϕ in (20) guarantees that $v_t = 1$ for all t. **Q.E.D.**

B. Description of the series

The main data source is the World Bank Development Indicators database. The series cover from 1960 to 2000. The variables used are:

- Real output: GDP, constant LCU (NY.GDP.MKTP.KN)
- Nominal output: GDP, current LCU (NY.GDP.MKTP.CN)
- Prices: GDP deflator (NY.GDP.DEFL.ZS)
- Money: Money, current LCU (FM.LBL.MONY.CN)
- Population: Population, total (SP.POP.TOTL)

The measure of money is the sum of currency outside banks and demand deposits other than those of central government. When data was not available for some country, the International Monetary Fund, International Financial Statistics was used.

- Real output: GDP, at constant prices (99BP.. or 99BR.. or 99BA..)
- Nominal output: GDP (99BZF or 99BCZF)
- Prices: GDP deflator (64ZF)
- Money: Money (34ZF) or M1 national definition (39M.. or 59M..)

Velocity of money is computed as nominal output divided by the money stock.

The countries in the sample by geographical areas are:

- Europe: Austria, Denmark, Greece, Iceland, Malta, Netherlands, Norway, Spain, Sweden, and Switzerland.

- America: Argentina, Bahamas, Barbados, Bolivia, Brazil, Canada, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Suriname, Trinidad and Tobago, United States, Uruguay, and Venezuela.
- Asia-Oceania: Australia, Fiji, India, Japan, Korea, Malaysia, Myanmar, New Zealand, Pakistan, Philippines, Singapore, Sri Lanka, and Thailand.
- Africa-Middle East: Algeria, Benin, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Congo, Cote d'Ivoire, Egypt, Gabon, Gambia, Ghana, Kenya, Madagascar, Malawi, Mali, Malta, Mauritania, Mauritius, Morocco, Nigeria, Rwanda, Saudi Arabia, Senegal, Sierra Leone, South Africa, Sudan, Syrian Arab Republic, Togo, and Tunisia,

For purposes of computations, the group of industrial countries is composed of the European countries excluding Malta and including Australia, Canada, Japan, New Zealand, and the United States. Latin-American countries are those of America without Canada and the United States. Asian countries are those of Asia-Oceania without Australia, Japan and New Zealand. Finally the group of African countries are those of Africa-Middle East including Malta.

C. Estimations of ϕ for individual countries

This Appendix reports the estimations of the transaction cost parameter (ϕ) for individual countries. This parameter is measured as percentage of nominal GDP. Countries are clustered in four groups according to the value of the parameter and ordered alphabetically within each group.

Group 1 (average ϕ = 0.12):

Australia (0.0937), Austria (0.1121), Bahamas (0.0546), Barbados (0.1333), Benin (0.1149), Burkina Faso (0.0607), Burundi (0.0885), Cameroon (0.0596), Canada (0.1132), Central African Rep. (0.1030), Chad (0.0352), Colombia (0.1840), Rep. Congo (0.1005), Costa Rica (0.2053), Cote d'Ivoire (0.1557), Dominican Republic (0.1301), Ecuador (0.2086), El Salvador (0.0730), Fiji (0.1086), Gabon (0.0840), Gambia (0.1424), Greece (0.1970), Guatemala (0.0638), Haiti (0.0858), Honduras (0.0891), Iceland (0.0790), India (0.1411), Jamaica (0.1688), Kenya (0.1594), Korea (0.1036), Madagascar (0.1526), Malawi (0.1098), Malaysia (0.1880), Mali (0.1321), Mauritania (0.0765), Mauritius (0.1692), Mexico (0.1417), New Zealand (0.0840), Nigeria (0.1304), Panama (0.0348), Paraguay (0.0762), Philippines (0.0657), Rwanda (0.0561), Saudi Arabia (0.1400), Senegal (0.0877), Sierra Leone (0.1480), South Africa (0.2621), Sri Lanka (0.1066), Thailand (0.0690), Togo (0.1655), Trinidad and Tobago (0.0673), United States (0.1095), Uruguay (0.2243), Venezuela (0.1884).

Group 2 (average $\phi = 0.40$):

Denmark (0.3111), Egypt (0.5382), Ghana (0.2712), Guyana (0.3305), Japan (0.5182), Morocco (0.6367), Myanmar (0.3887), Norway (0.3006), Pakistan (0.5079), Singapore (0.3071), Spain (0.5152), Sudan (0.3078), Switzerland (0.3719), Tunisia (0.3573).

Group 3 (average $\phi = 1.41$):

Algeria (1.3767), Argentina (1.0438), Bolivia (1.2546), Brazil (2.1980), Malta (1.5161), Netherlands (0.9278), Peru (2.2353), Suriname (1.3360), Sweden (1.2170), Syrian Arab Republic (0.9849).

Group 4 (average $\phi = 14.7435$):

Nicaragua (14.7435).

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Table 1
Correlations between the growth rate of money and...

	Number of observations	… inflation		… velocity	
		Averages	Std. dev.	Averages	Std. dev.
All countries	79	0.9885	0.9662	0.2275	0.4066
Industrial	14	0.9305	0.6277	0.6645	0.7334
Latin-American	24	0.9868	0.9625	0.1155	0.5243
Asian	10	0.8540	0.4821	0.4131	0.2817
African	31	0.9666	0.5660	0.2131	0.5476

Note: Data is annual and covers the period 1960-2000. Money is M1, inflation is the growth rate of the GDP deflator and velocity is nominal GDP divided by the money stock. Details on the countries used as well as on geographical areas and variable definitions are provided in Appendix B.

Table 2
Regressions for velocity

	Average		Std. dev.	
Variable	(1)	(2)	(3)	(4)
Constant	7.7002	7.5404	1.0631	1.0537
	(16.5)	(18.6)	(3.77)	(4.09)
Money	0.5495	3.0688	0.1422	0.3801
	(1.57)	(5.29)	(3.70)	(5.52)
Output	-45.577	-40.948	11.274	12.691
	(-2.37)	(-2.46)	(1.93)	(2.37)
ϕ	-	-140.84	-	-47.289
		(-5.10)		(-4.01)
R^2	0.115	0.343	0.204	0.345
Observations	79	79	79	79

Note: Results of regressions of velocity, as measured by nominal GDP divided by M1, for a cross section of 79 countries. Column (1) regress average velocity on a constant, the average growth rate of money and the average growth rate of output where the averages are taken over the period 1960-2000 using annual data. Column (2) adds the estimation of ϕ to the regressors. Column (3) regress the standard deviation of velocity computed for each country in the cross section on the standard deviations of the growth rate of money and output. Column (4) adds the estimation of ϕ to the regressors in (3). *T*-statistics appear in parenthesis below the point estimates.

Table 3
Regressions for inflation

	Average		Std. dev.	
Variable	(1)	(2)	(3)	(4)
Constant	-0.0213	-0.0174	-0.0883	-0.0784
	(-0.73)	(-0.60)	(-0.59)	(-0.95)
Money	1.2497	1.1877	0.6588	0.4076
	(56.6)	(28.5)	(32.4)	(18.6)
Output	-1.8572	-1.9710	1.8727	0.3773
	(-1.54)	(-1.65)	(0.61)	(0.22)
ϕ	-	3.4621	-	49.924
		(1.75)		(13.3)
R^2	0.978	0.979	0.934	0.980
Observations	79	79	79	79

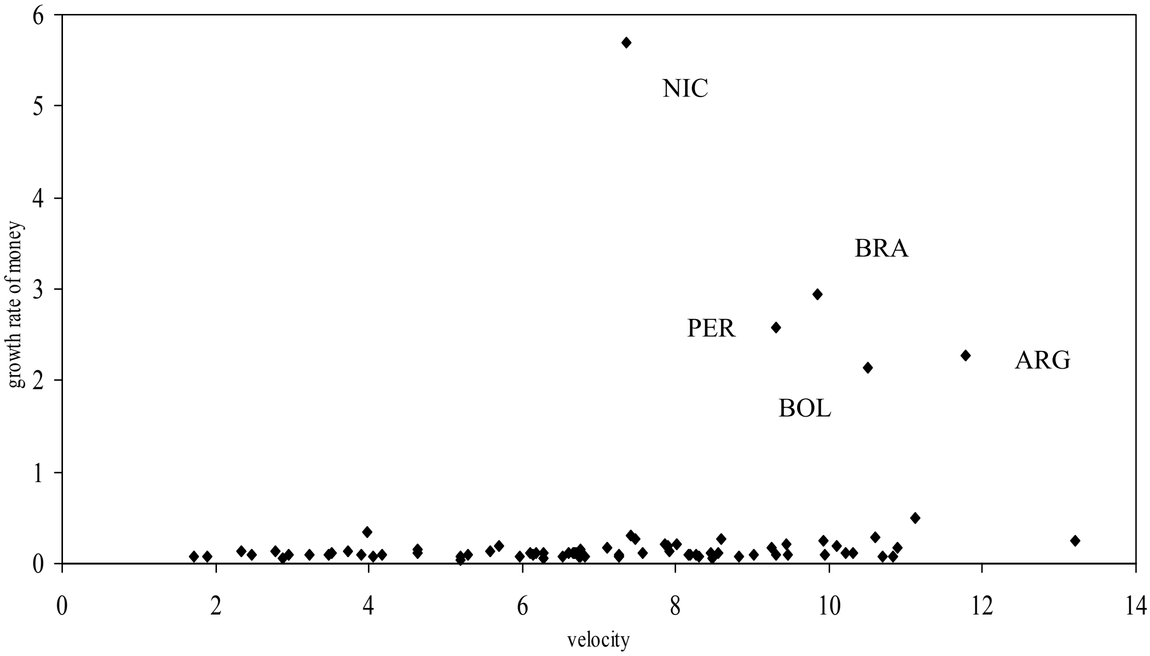
Note: Results of regressions of inflation, as measured by the growth rate of the GDP deflator, for a cross section of 79 countries. Column (1) regress average inflation on a constant, the average growth rate of money and the average growth rate of output where the averages are taken over the period 1960-2000 using annual data. Column (2) adds the estimation of ϕ to the regressors. Column (3) regress the standard deviation of inflation computed for each country in the cross section on the standard deviations of the growth rate of money and output. Column (4) adds the estimation of ϕ to the regressors in (3). *T*-statistics appear in parenthesis below the point estimates.

Table 4
Correlations between the growth rate of money and...

	Number of observations	Average ϕ	… inflation		… velocity	
			Averages	Std. dev.	Averages	Std. dev.
All countries	79	0.0052	0.9885	0.9662	0.2275	0.4066
Group 1	54	0.0012	0.9759	0.7074	0.4829	0.5377
Group 2	14	0.0040	0.9842	0.8938	0.9432	0.8132
Group 3	10	0.0141	0.9579	0.9363	0.9549	0.8516

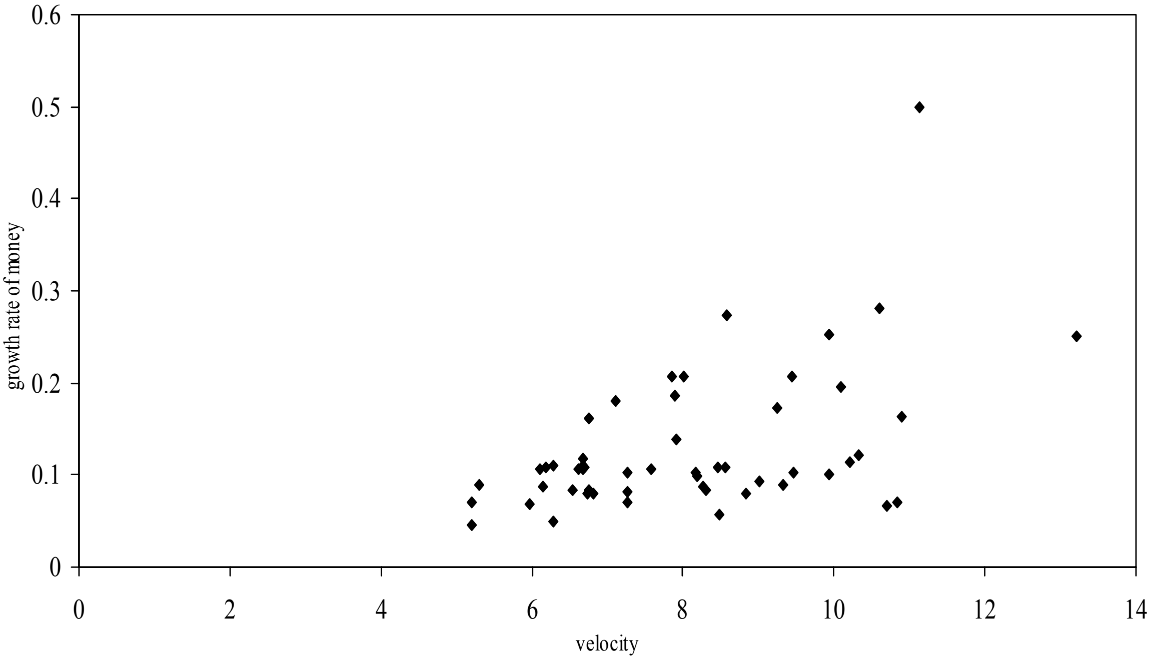
Note: Data is annual and covers the period 1960-2000. Money is M1, inflation is the growth rate of the GDP deflator and velocity is nominal GDP divided by M1. Details on the countries used as well as on the definitions of the groups and variable definitions are provided in Appendices B and C.

Figure 1: Velocity vs. Growth Rate of Money (all countries)



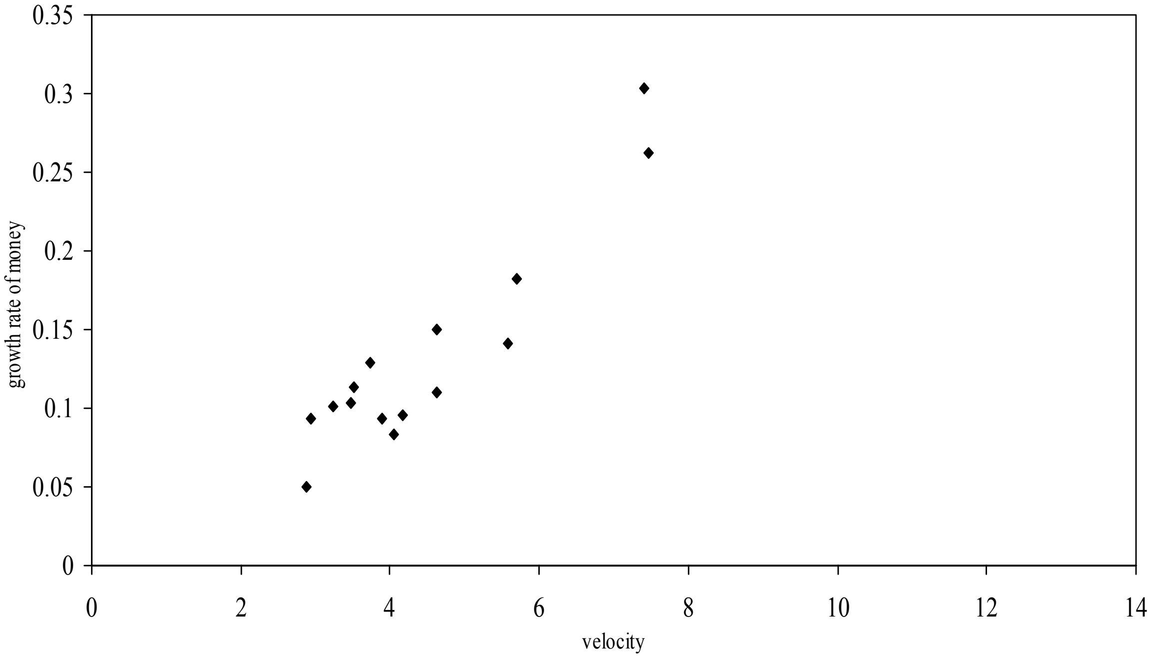
Note: Money is defined as M1. Velocity is nominal GDP divided by the money stock. Each dot represents the combination of the average velocity and the average growth rate of money computed from annual data over the period 1960-2000 for a particular country. See Appendix A for definitions of variables and the list of countries.

Figure 2: Velocity vs. Growth Rate of Money (group 1)



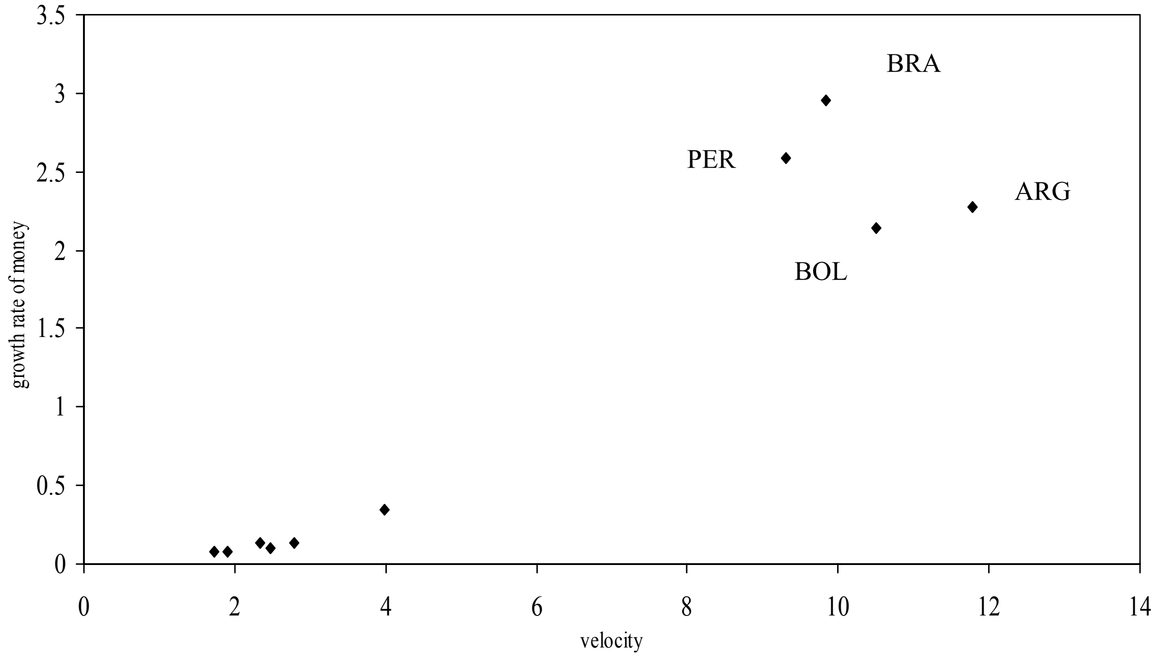
Note: Money is defined as M1. Velocity is nominal GDP divided by the money stock. Each dot represents the combination of the average velocity and the average growth rate of money computed from annual data over the period 1960-2000 for a particular country. See Appendix A and C for definitions of variable as well as the list of countries and groups.

Figure 3: Velocity vs. Growth Rate of Money (group 2)



Note: See note on Figure 2.

Figure 4: Velocity vs. Growth Rate of Money (group 3)



Note: See note on Figure 2.